**Introduction**

My teaching activities at the university began with a classical lecture course of the "*equation of mathematical physics*" for the students in the specialty "*mechanics*". Given their professional interests, I began the course with practical applications. I wanted to show them that the equations of mathematical physics are of interest to them, first of all, because they are mathematical models of the nature phenomena.

We have the clear scheme. The elementary volume of the region is chosen. Starting from the laws of physics, we estimate the difference between what enters and goes into this volume over a certain time interval. Then the passage to the limit is realized, when this volume is compressed into a point, and the length of the given time interval tends to zero. As a result, one or another equation of mathematical physics is obtained... Many years later the question arose as to how justified this limiting transition is?

Norbert Wiener once wrote that the most productive for the development of the sciences are the areas relating to the "nobody's territory" between the different established sciences. Perhaps, this statement in the best degree characterizes the nature of the problem under consideration. On the one hand, the construction of a mathematical model of the physical process is really a translation of the specific laws of physics into the mathematical language. This is the business of physics. On the other hand, the passage to the limit is one of the most important mathematical procedures, to which it is customary to treat with the utmost care. The justification of the limit transitions is mathematical operation.

The majority of mathematicians, specialists in the equations of mathematical physics perceive a particular equation as a subject of research, given in a finished form. It is nice that this object has a certain physical sense. This can serve as a justification for the practical significance of the mathematical research. However, the main thing is the analysis itself, in particular, the existence of a solution in one sense or another under certain restrictions on the parameters of the problem, its uniqueness, smoothness, qualitative behavior, the properties of the dependence of the solution on parameters, etc. But the process of determination of the equation, according to many mathematicians, refers to physics...

Most physicists agree with this. However, they pay the greatest attention to the evaluation of physical factors that determine the corresponding balance relations in the chosen elementary volume. The justification of the convergence is not relevant to their most important scientific interests. Therefore, they usually either simply formally pass to the limit without thinking about the degree of validity of this procedure (the reasoning of the convergence is not theirs, but of mathematicians) or they assume that the functions under consideration are sufficiently smooth, as a result of which the required limit actually exists.

It would seem that the problem is solved. However, a new question arises, how do we know that the corresponding functions have this degree of smoothness? The functions in question are precisely those unknowns with respect to which the state equations are obtained. Further, it would seem, everything is simple. The functional properties of the equations of mathematical physics are established by means of a fairly well-developed theory of certain equations. We use these results and obtain the desired properties of their solutions!

However, the last question arises. The above results of the theory of equations of mathematical physics have the following form: under certain constraints on the parameters of a given equation, its solution has the corresponding properties. Here it is assumed that the equation has already given. However, how can we establish the properties of the solution of the equation in a situation, where the equation has not yet been obtained?

Thus, we have to two obviously true statements. If a function has a proper degree of smoothness, then it satisfies the corresponding equation. If the equation is given, then (under certain suppositions) its solution has a proper degree of smoothness. However, these statements do not solve the above problem.

Over time, I had a desire to somehow understand this situation. Then there was a special course, which I lectured for many years at the Mechanics and Mathematics Faculty of the al-Farabi Kazakh National University. Then a small book in Russian appeared (see Sequential models of mathematical physics. – Almaty, Print-S, 2004. – 164 p.). Its main idea is described in the paper Sequential models of physical phenomenon and justification of mathematical modeling // Advanced Mathematical Models & Applications. Vol. 2. No. 1, 2017. – PP. 6–13.

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The general structure of the book is given in the figure below. In the first chapter we give a general formulation of the problem, based on the concept of the classical solution of the problem of mathematical physics. The subject of the following chapter is a generalized solution of the problem of mathematical physics. Its connection with the classical solution is analyzed, and its independence from the classical solution at the stage of determination and practical implementation is shown. However, this concept has the same drawback, and the question of the degree of validity of the passage to the limit in the derivation of the mathematical model remains open.

These chapters constitute the first part of the book. Its second part, which includes three chapters, is devoted to the general problem of justifying the convergence. The third chapter describes different forms of sequence convergence. This is based on an estimate of the degree of proximity of an element of a sequence with large enough number to an element called the limit. The practical application of these definitions is difficult if the value of the limit and even the fact of convergence remain unknown. In mathematical analysis, at this step of analysis, the Cauchy convergence criterion is used, according to which any fundamental sequence is convergent. The sequence is fundamental if its elements approach each other unboundedly. Checking this condition requires knowing only the elements of the sequence, which is acceptable in practice.

The fourth chapter begins with an analysis of examples of spaces, where the Cauchy criterion does not work, and hence the fundamental sequence may not converge. Thus, all spaces are divided into complete and incomplete, depending on whether the Cauchy criterion it is true or not. We would like to prove the convergence for the incomplete spaces too. The way to overcome this difficulty is based on Cantor's definition of real numbers. There are equivalence classes of fundamental sequences of rational numbers. In this case, the divergent fundamental sequence of rational numbers determines an irrational number, and different these sequences can define the same real number. Such sequences are called equivalent. It is important that any fundamental sequence real numbers converges, and any real number (the object of an extended set) can be arbitrarily accurately approximated by rational numbers (objects of the original set).

In the fifth chapter, we show that the objects defined in this way have all the properties that are assumed to be attributed to real numbers. Then the results obtained are extended to the general case. A theorem on the completion of a metric space is considered. By this result, any incomplete metric space can be extended to a complete space (its completion) so that any element of the completion can be arbitrarily closely approximated by the elements of the original set. This is the basis of the sequential method. Now we consider sequential objects that are classes of equivalent fundamental sequences.

The third part gives an overview of different sequential objects. In three of its chapters we consider *p*-adic numbers, sequential controls, and distributions. The last example is especially important, since a generalized approach in mathematical physics is based on the theory of distributions.

The final part includes a unique chapter. Here, a sequential method is used to determine the sequential form of the model. We consider classes of equivalent fundamental sequences of "approximate solutions" of the problem. Obtaining this result does not require any a priori supposition for the solution of the problem. It is characteristic that under additional restrictions on the parameters of the problem, the fundamental sequences under consideration converge to a function satisfying the integral identity underlying the generalized solution of the problem. This proves the justification of the generalized approach in problems of mathematical physics. The result established in the first section, according to which a sufficiently smooth generalized solution of the problem is its classical solution, makes it possible to substantiate the classical approach too. Thus, the problem initially posed is completely solved. Extremely important is the fact that the same algorithm is applied to the practical solving of the classical, generalized and sequential models of the system. Depending on the constraints on the parameters of the system, it can output to one or another form of the model.

We would like to simplify the technical calculations as much as possible without losing the essence of the problem. In this connection, we confine ourselves to considering the single extremely simple illustrative example. This is a stationary heat transfer phenomenon for the one-dimensional case.

Note that in the process of answering one particular question, one has to turn to different questions of mathematical and mathematical physics, the theory of differential equations and computational mathematics, the theory of numbers and the theory of distributions, algebra and topology, mathematical analysis and the theory of optimal control. Thus, this book can serve as an illustration of the unity of mathematics, as well as the absence of clear boundaries between mathematics and physics. The book is recommend for a wide range of specialists and students of mathematical and physical specialties.

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